Number · Algebra · Ratio, proportion and rates of change · Geometry & measures · Probability · Statistics

BIDMAS

...or BODMAS. Use the correct order of operations; take care when using a calculator.

- **B**rackets
- **I**ndices (or p**O**wers)
- **D**ivision and **M**ultiplication
- **A**ddition and **S**ubtraction

Types of number

Integer: a "whole" number
Factors; the divisors of an integer
➔ **Factors of 12 are 1, 2, 3 ,4, 6, 12**
Multiples; a "times table" for an integer (will continue indefinitely)
➔ **Multiples of 12 are 12, 24, 36 ...**
Prime number: an integer which has exactly two factors (1 and the number itself). Note: 1 is not a prime number.

HCF, LCM

Highest **C**ommon **F**actor (HCF)
➔ **Factors of 6 are 1, 2, 3, 6**
Factors of 9 are 1, 3, 9
HCF of 6 and 9 is 3
Lowest **C**ommon **M**ultiple (LCM)
➔ **Multiples of 6 are 6, 12, 18, 24, ...**
Multiples of 9 are 9, 18, 27, 36, ...
LCM of 6 and 9 is 18

Prime factors

Write a number as a product of its prime factors; use indices for repeated factors:
➔ $720 = 5 \times 3^2 \times 2^4$

Powers and roots

Special indices: for any value a:

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

➔ $3^{-4} = \frac{1}{3^4} = \frac{1}{81}$

Calculating with fractions

Adding or subtracting fractions; use a common denominator...
➔ $\frac{4}{5} - \frac{1}{3} = \frac{12}{15} - \frac{5}{15} = \frac{7}{15}$
Multiplying fractions; multiply numerators and denominators...
➔ $\frac{4}{7} \times \frac{2}{3} = \frac{8}{21}$
Dividing fractions; "flip" the second fraction, then multiply...
➔ $\frac{2}{7} \div \frac{5}{6} = \frac{2}{7} \times \frac{6}{5} = \frac{12}{35}$

Fractions, decimals

Fraction is numerator ÷ denominator
➔ $\frac{5}{8} = 5 \div 8 = 0.625$
Use place values to change decimals to fractions. Simplify where possible.
➔ $0.45 = \frac{45}{100} = \frac{9}{20}$
Learn the most frequently used ones:

$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{4}$
0.5	0.25	0.1	0.2	0.75

Surds

Look for the biggest square number factor of the number:
➔ $\sqrt{80} = \sqrt{16 \times 5} = 4\sqrt{5}$

Standard form

N9

Standard form numbers are of the form $a \times 10^n$ where $1 \le a < 10$ and n is an integer.

Standard units

1 tonne = 1000 kilograms
1 kilogram = 1000 grams

1 kilometre = 1000 metres
1 metre = 100 centimetres
= 1000 millimetres
1 centimetre = 10 millimetres

1 day = 24 hours
1 hour = 60 minutes = 3600 seconds
1 minute = 60 seconds

Rounding

Truncate the number, then use a "decider digit" to round up or down.
Decimal places: use the decimal point
➔ **162.3681 to 2dp;**
162.36 | 81 **= 162.37 to 2dp**
Significant figures: use the first non-zero digit.
➔ **162.3681 to 2sf;**
16 | 2.3681 **= 160 to 2sf**
➔ **0.007 039 to 3sf;**
0.007 03 | 9 **= 0.007 04 to 3sf**

Error intervals

Find the range of numbers that will round to a given value:
➔ $x = 5.83$ **(2 decimal places)**
$5.825 \le x < 5.835$
➔ $y = 46$ **(2 significant figures)**
$45.5 \le y < 46.5$
Note use of ≤ and <, and that the last significant figure of each is 5

Algebraic notation

$$ab = a \times b$$
$$3y = y + y + y$$
$$a^2 = a \times a$$
$$a^3 = a \times a \times a$$
$$a^2b = a \times a \times b$$
$$\frac{a}{b} = a \div b$$

Equations and identities

An equation is true for some particular value of x
➔ $2x + 1 = 7$ **is true if** $x = 3$
...but an identity is true for every value of x
➔ $(x + a)^2 \equiv x^2 + 2ax + a^2$
(note the use of the symbol ≡)

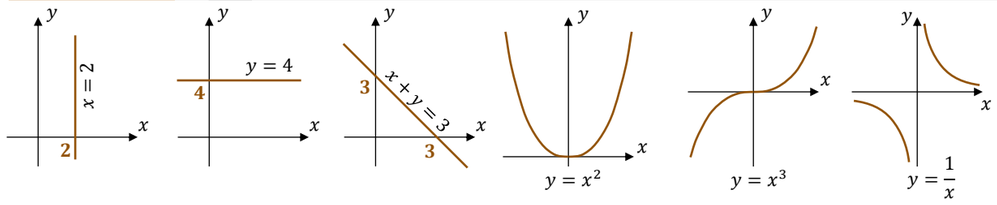
Laws of indices

For any value a:

$$a^x \times a^y = a^{x+y}$$
$$\frac{a^x}{a^y} = a^{x-y}$$
$$(a^x)^y = a^{xy}$$

➔ $\left(\frac{2pq^4}{p^3q}\right)^3 = \frac{8p^3q^{12}}{p^9q^3} = \frac{8q^9}{p^6}$ or $8q^9p^{-6}$

Standard graphs



$y = mx + c$

Equation of straight line $y = mx + c$
m is the gradient; c is the y intercept:
➔ **Find the equation of the line that joins (0 , 3) to (2 , 11)**
Find its gradient...

$$\frac{11 - 3}{2 - 0} = \frac{8}{2} = 4$$

...and its y intercept...
Passes through (0 , 3), so c = 3
Equation is $y = 4x + 3$

Parallel lines: gradients are equal;
➔ $y = 2x + 3$ **and** $y = 2x - 5$ **both have gradient 2 so are parallel.**

Expanding brackets

$$p(q + r) = pq + pr$$
➔ $5(x - 2y) = 5x - 10y$
$(x + a)(x + b) = x^2 + ax + bx + ab$
➔ $(2x - 3)(x + 5)$
$= 2x^2 - 3x + 10x - 15$
$= 2x^2 + 7x - 15$
Reverse of expanding is factorising - putting an expression into brackets.

Quadratics

Solve a quadratic by factorising.
➔ **Solve** $x^2 - 8x + 15 = 0$
Put into brackets (taking care with any negative numbers)...
$(x - 3)(x - 5) = 0$
...then either $x - 3 = 0$ or $x - 5 = 0$
so that $x = 3$ **or** $x = 5$.

Difference of two squares

$$a^2 - b^2 = (a + b)(a - b)$$
➔ $x^2 - 25 = (x + 5)(x - 5)$

Simultaneous equations

➔ **Solve** $\begin{cases} 2x + 3y = 11 \\ 3x - 5y = 7 \end{cases}$
Multiply to match a term in x or y
$\begin{cases} 10x + 15y = 55 \\ 9x - 15y = 21 \end{cases}$
Add or subtract to cancel...
$19x = 76$, **so** $x = 4$
Finally, substitute and solve...
$2 \times 4 + 3y = 11$, **so** $y = 1$

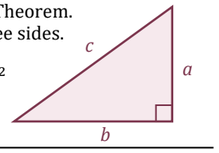
Rearrange a formula

The subject of a formula is the term on its own. Use rules that "balance" the formula to change its subject
➔ **Make** x **the subject of**
$2x + 3y = z$
Here, subtract $3y$ from both sides...
$2x = z - 3y$
...then divide both sides by 2

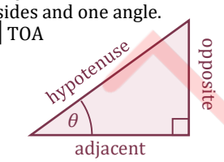
$$x = \frac{z - 3y}{2}$$

Right angled triangles

Pythagoras Theorem.
Links all three sides.
No angles.
$a^2 + b^2 = c^2$



Trigonometry.
Links two sides and one angle.
SOH | CAH | TOA



$\sin\theta = \frac{\text{opp}}{\text{hyp}}$ $\cos\theta = \frac{\text{adj}}{\text{hyp}}$ $\tan\theta = \frac{\text{opp}}{\text{adj}}$

Use "2ndF" or "SHIFT" key to find a missing angle

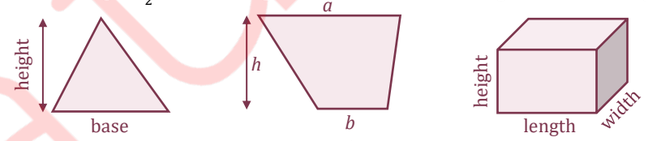
The longest side of any right angled triangle is the hypotenuse; check that your answer is consistent with this.

Special values of sin, cos, tan
Learn (or be able to find without a calculator)...

$\theta°$	$\sin\theta°$	$\cos\theta°$	$\tan\theta°$
0	0	1	1
30	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90	1	0	

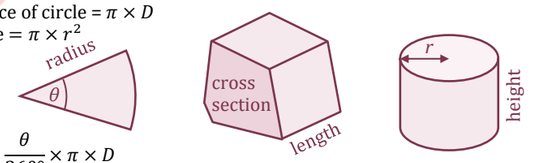
Areas and volumes

Area of triangle = $\frac{1}{2} \times$ base $\times$ height
Volume of cuboid = length × width × height



Area of trapezium = $\frac{1}{2}(a + b) \times h$

Circumference of circle = $\pi \times D$
Area of circle = $\pi \times r^2$



Arc length = $\frac{\theta}{360°} \times \pi \times D$
Area of sector = $\frac{\theta}{360°} \times \pi \times r^2$
Volume of cylinder = $\pi r^2 \times$ height
Volume of prism = area of cross section × length

Transformations

Reflection
- Line of reflection

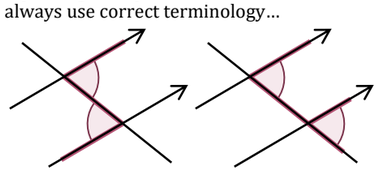
Translation
- Vector

Rotation
- Centre of rotation
- Angle of rotation
- Clockwise or anticlockwise

Enlargement
- Centre of enlargement
- Scale factor (if SF < 1 the shape will get smaller).

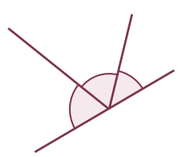
Angle facts

Equal angles in parallel lines: always use correct terminology...

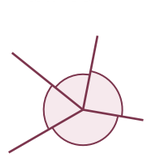


Alternate angles Corresponding angles

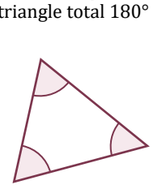
Angles on a straight line total 180°



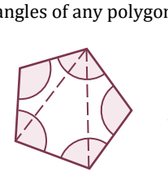
Angles in a full turn total 360°



Interior angles in a triangle total 180°

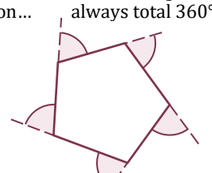


Use this for the interior angles of any polygon...



...or $180° \times (n - 2)$

Exterior angles always total 360°



Sequences

Triangular numbers:

1st	2nd	3rd	4th	5th
1	3	6	10	15

Square numbers ($n^2 = n \times n$):

1^2	2^2	3^2	4^2	5^2
1	4	9	16	25

Cube numbers ($n^3 = n \times n \times n$):

1^3	2^3	3^3	4^3	5^3
1	8	27	64	125

nth term of an arithmetic (linear) sequence is $an + d$
➔ n**th term of 5, 8, 11, 14, ... is** $3n+2$ **(always increases by 3 first term is** $3 \times 1 + 2 = 5$**)**
Geometric sequence; multiply each term by a constant ratio
➔ **3, 6, 12, 24, ... (ratio is 2)**
Fibonacci sequence; make the next term by adding the previous two ...
➔ **2, 4, 6, 10, 16, 26, 42, ...**

Probability

$$p = \frac{n(\text{equally likely favourable outcomes})}{n(\text{equally likely possible outcomes})}$$

$p = 0$	impossible
$0 < p < 0.5$	unlikely
$p = 0.5$	evens
$0.5 < p < 1$	likely
$p = 1$	certain

Probability rules

Multiply for independent events
➔ **P(6 on dice and H on coin)**

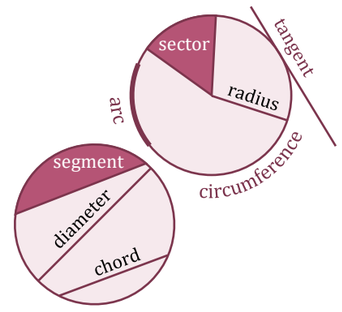
$$\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$$

Add for mutually exclusive events
➔ **P(5 or 6 on dice)**

$$\frac{1}{6} + \frac{1}{6} = \frac{2}{6}$$

Apply these rules to tree diagrams.

Parts of a circle



Division using ratio

Use a ratio for unequal sharing
➔ **Divide £480 in the ratio 7 : 5**
7 + 5 = 12, then £480 ÷ 12 = £40
7 × £40 = £280, 5 × £40 = £200
(check: £280 + £200 = £480 ✓)

Ratio and fractions

Link between ratios and fractions
➔ **Boys to girls in ratio 2 : 3**
$\frac{2}{5}$ **are boys,** $\frac{3}{5}$ **are girls.**

Percentages

$$y \text{ percent of } x = \frac{y}{100} \times x$$

➔ **Increase £58 by 26%.**
$\frac{26}{100} \times £58 = £15.08$
£58 + £15.08 = £73.08

$$y \text{ as a percentage of } x = \frac{y}{x} \times 100\%$$

➔ **The population of a town increases from 3500 to 4620**
Find the percentage increase.
$\frac{1120}{3500} \times 100\% = 32\%$
Note: fraction $= \frac{\text{increase}}{\text{original}}$
Learn the most frequently used ones:

$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{100}$
50%	25%	10%	20%	1%

Speed, distance, time

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

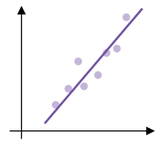
➔ **A car travels 90 miles in 1 hour, 30 minutes. Find its average speed.**
90 miles ÷ 1.5 hours = 60 mph

Averages

Mode: most frequently occurring
Median: put the data in numerical order, then choose the middle one
Mean $= \frac{\text{total of items of data}}{\text{number of items of data}}$

Correlation

Positive correlation



Negative correlation