

UKMT IMC

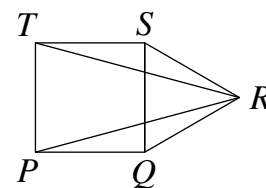
Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **60 minutes**.
No answers, or personal details, may be entered after the allowed time is over.
3. The use of blank or lined paper for rough working is allowed; **squared paper, calculators and measuring instruments are forbidden**.
4. **Use a B or an HB non-propelling pencil**. Mark at most one of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
5. **Do not expect to finish the whole paper in the time allowed**. The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
6. **Scoring rules:**
5 marks are awarded for each correct answer to Questions 1-15;
6 marks are awarded for each correct answer to Questions 16-25;
Each incorrect answer to Questions 16-20 loses 1 mark;
Each incorrect answer to Questions 21-25 loses 2 marks.
7. **Your Answer Sheet will be read by a machine**. Do not write or doodle on the sheet except to mark your chosen options. The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of eraser stuck to the page, the machine will interpret the mark in its own way.
8. **The questions on this paper are designed to challenge you to think, not to guess**. You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.
9. To accommodate candidates sitting at other times, please do not discuss the paper on the internet until **12 noon GMT on Friday 31st January**, when the solutions video will be released at ukmt.org.uk/competition-papers. Candidates in time zones more than 5 hours ahead of GMT must sit the paper on Thursday 30th January (as defined locally).

1. Which one of the following expressions has a value closest to 0?
 A $2 \times 5 - 8 \times 3$ B $3 \times 4 - 7 \times 4$ C $4 \times 3 - 6 \times 5$ D $5 \times 2 - 5 \times 6$ E $6 \times 1 - 4 \times 7$
2. What is the remainder when 2 652 134 is divided by 13?

A 1 B 2 C 3 D 4 E 5

3. The diagram shows a square $PQST$ and an equilateral triangle QRS .
 What is the size, in degrees, of angle PRT ?



A 10 B 15 C 20 D 25 E 30

4. All Felix's cats are normal cats. Together, they have 12 more legs than they have tails.
 In total, how many ears do they have?

A 2 B 4 C 6 D 8 E 10

5. What is $5 \div (((5 \div 5) \div (5 \div 5)) \div 5)$?

A $\frac{1}{25}$ B $\frac{1}{5}$ C 1 D 5 E 25

6. Owen chooses a positive integer n so that $3n + 7$ is an even integer.
 Which of these is an odd integer?

A $n - 1$ B $n + 3$ C $3n + 2$ D $4n$ E $5n + 3$

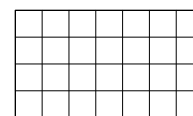
7. "I can swim faster than you," said the dolphin to the shark.
 "That is not true," said the shark to the dolphin.
 "You are both wrong," said the octopus to the dolphin and the shark.
 "You are right," said the starfish to the octopus.
 How many of the dolphin, shark, octopus and starfish were telling the truth?

A 0 B 1 C 2 D 3 E 4

8. What is the value of $34\frac{1}{7} \div 17\frac{1}{14}$?

A 2 B $2\frac{1}{17}$ C $2\frac{1}{14}$ D $2\frac{1}{7}$ E $2\frac{1}{2}$

9. Lottie wants to colour two rows and two columns of the 4×7 grid shown so that the coloured rows do not touch and also the coloured columns do not touch.



In how many ways can she do this?

A 45 B 42 C 40 D 35 E 30

10. What is the value of $\sqrt{2025^2 - 2024} - 2025$?

A 2022 B 2023 C 2024 D 2025 E 2026

11. Emily multiplied two integers together. Her answer was 360. Finley increased each of Emily's integers by one and then multiplied the two new integers together. His answer was 400.
 What was the sum of Emily's two integers?

A 39 B 40 C 42 D 45 E 48

12. Jane's farm is home to many cats and dogs. At the start of the week the ratio of cats to dogs was 3 : 5. Then 32 cats, but no dogs, arrived and the ratio became 5 : 3. How many dogs are there on Jane's farm?

A 12 B 18 C 24 D 25 E 30

13. The product of the first three numbers in the box below is equal to the product of the last three numbers. What is the value of x ?

| | | | | |
|-----|-----|-----|-----|----|
| x | 120 | 496 | 360 | 48 |
|-----|-----|-----|-----|----|

A 72 B 84 C 96 D 128 E 144

14. What is the sum of the recurring decimals $0.\dot{1} + 0.\dot{2} + 0.\dot{3} + 0.\dot{4}$?

A $1.\dot{1}$ B $1.\dot{1}\dot{0}$ C 1.1 D $1.\dot{0}\dot{1}$ E 1

15. What is the value of $\frac{3^6 - 3^4}{2^9 - 2^3}$?

A $\frac{9}{32}$ B $\frac{9}{16}$ C $\frac{9}{12}$ D $\frac{9}{7}$ E $\frac{9}{5}$

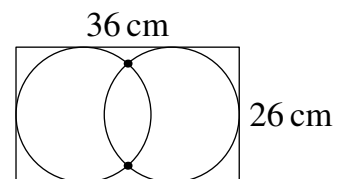
16. David likes books. He only ever pays 99p or £1.99 for an electronic book. In the last few months he has spent £56.56 on electronic books. How many electronic books costing £1.99 has he bought in that time?

A 8 B 13 C 15 D 17 E 23

17. Dark green paint is made by mixing blue and yellow paint such that 60% is blue. Light green paint is made by mixing blue and yellow paint such that 60% is yellow. Pablo mixes dark and light green paint such that 60% of the mixture is dark paint. What is the ratio of blue to yellow in Pablo's paint?

A 2 : 3 B 12 : 13 C 1 : 1 D 13 : 12 E 3 : 2

18. Two circles each touch three sides of a rectangle measuring 36 cm \times 26 cm, as is shown in the diagram, which is not drawn to scale. What is the distance between the two points where the circles intersect?



A 18 cm B 20 cm C 21 cm D 22 cm E 24 cm

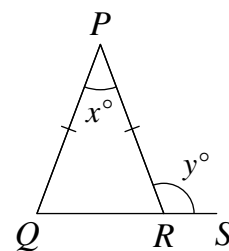
19. Rob, Rog and Roy all painted some fence posts one day. Rob painted 45, Rog painted 51 and Roy painted 48. One of them painted twice as many posts in the morning as he did in the afternoon, a second one painted three times as many in the morning as in the afternoon and the third one painted four times as many in the morning as in the afternoon. Who painted the most fence posts in the morning?

A Rob B Rog
 C Roy D Rob and Roy, who painted the same number
 E Rob, Rog and Roy, who all painted the same number

20. The triangle PQR is isosceles with $PQ = PR$. The point S lies on the extension of the line QR as shown.

$\angle RPQ = x^\circ$ and $\angle SRP = y^\circ$, where x, y and $\frac{y}{x}$ are integers.

What is the largest possible value of $\frac{y}{x}$?



- A 36 B 23 C 17 D 8 E 5

21. What is the remainder when 8^8 is divided by 5?

- A 0 B 1 C 2 D 3 E 4

22. A sphere of radius 4 cm and a sphere of radius 16 cm are placed on a horizontal surface so that they touch. What is the distance, in cm, between the points where the spheres touch the surface?

- A 12 B 14 C $\sqrt{240}$ D 16 E 20

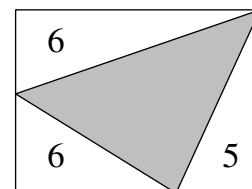
23. The point P is inside a rectangle. The distance from P to one corner of the rectangle is 5 cm and its distance from the opposite corner is 14 cm. The distance from P to a third corner is 10 cm. What is the distance, in cm, from P to the fourth corner?

- A $\sqrt{29}$ B 9 C 10 D 11 E $\sqrt{171}$

24. Two white roses and a yellow rose costs £5. Two white roses and three red roses cost £10.50. Three yellow roses and two red roses cost £11. What would be the total cost of one red rose, one white rose and one yellow rose?

- A £5 B £5.50 C £6 D £6.50 E £7

25. The diagram shows a rectangle which has been divided into four triangles. The areas, in cm^2 , of three of the triangles are as shown. What is the area of the shaded triangle?



- A 11 cm^2 B 13 cm^2 C 14 cm^2 D 15 cm^2 E 16 cm^2